Continuity and Differentiability

Subjects : Maths **Previous Year Questions**

Maths - Section A (MCQ)

1. Let f be a real polynomial of degree n such that $f(x) = f'(x)f''(x)$, for all $x \in \mathbb{R}$. If $f(0) = 0$, then $36\left(f'(2) + f''(2) + \int_0^2 f(x)\,dx\right)$ is equal to:

A) 42 **B)** 46 **C)** 56 **D)** 66

2. For the function $f(x) = e^{\sin|x|} - |x|$, $x \in \mathbb{R}$, consider the following statements:
Statement I: f is differentiable for all $x \in \mathbb{R}$.
Statement II: f is increasing in $\left(-\pi, -\frac{\pi}{2}\right)$.
In the light of the above statements, choose the correct answer from the options given below:

A) Both Statement I and Statement II are true

B) Both Statement I and Statement II are false

C) Statement I is true but Statement II is false

D) Statement I is false but Statement II is true

3. Let $f(x) = \begin{cases} \frac{1}{3}, & x \le \pi/2 \\ \frac{b(1-\sin x)}{(\pi - 2x)^2}, & x > \pi/2 \end{cases}$. If f is continuous at $x = \pi/2$, then the value of $\int_0^{3b-6} |x^2 + 2x - 3|\,dx$ is:

A) 5 **B)** 2 **C)** 3 **D)** 4

4. If $y = \tan^{-1}\left(\frac{3\cos x - 4\sin x}{4\cos x + 3\sin x}\right) + 2\tan^{-1}\left(\frac{x}{1+\sqrt{1-x^2}}\right)$, then $\frac{dy}{dx}$ at $x = \frac{\sqrt{3}}{2}$ is equal to:

A) 3 **B)** -1 **C)** 1 **D)** 2

5. Let $f(x) = \lim_{\theta \to 0}\left(\frac{\cos \pi x - x^{\left(\frac{2}{\theta}\right)} \sin(x-1)}{1 + x^{\left(\frac{2}{\theta}\right)}(x-1)}\right), x \in R.$
Consider the following two statements :
(I) $f(x)$ is discontinuous at $x = 1$.
(II) $f(x)$ is continous at $x = -1$. Then,

A) Neither (I) nor (II) is True **B)** Both (I) and (II) are True

C) Only (II) is True **D)** Only (I) is True

6. Let $\alpha, \beta \in \mathbb{R}$ be such that the function
$f(x) = \begin{cases} 2\alpha(x^2 - 2) + 2\beta x & , x < 1 \\ (\alpha + 3)x + (\alpha - \beta) & , x \ge 1 \end{cases}$
be differentiable at all $x \in \mathbb{R}$. Then $34(\alpha + \beta)$ is equal to

A) 84 **B)** 48 **C)** 36 **D)** 24

7. If the function $f(x) = \frac{e^x(e^{\tan x - x} - 1) + \log_e(\sec x + \tan x) - x}{\tan x - x}$ is continuous at $x = 0$, then the value of $f(0)$ is equal to

A) 2 B) $\frac{2}{3}$ C) $\frac{1}{2}$ D) $\frac{3}{2}$

8. Let [t] denote the greatest integer less than or equal to t. If the function

$$f(x)=\begin{cases} b^2\sin\left(\frac{\pi}{2}\left[\frac{\pi}{2}(\cos x+\sin x)\cos x\right]\right), & x<0 \\ \frac{\sin x-\frac{1}{2}\sin 2x}{x^3} & , x>0 \\ a & , x=0 \end{cases}$$

is continuous at $x=0$, then a^2+b^2 is equal to

A) $\frac{5}{8}$ B) $\frac{9}{16}$ C) $\frac{3}{4}$ D) $\frac{1}{2}$

9. Consider the following three statements for the function $f:(0,\infty)\rightarrow\mathbb{R}$ defined by $f(x)=|log_e x|-|x-1|$:
(I) f is differentiable at all $x>0$.
(II) f is increasing in (0, 1).
(III) f is decreasing in (1, ∞).
Then:

A) All (I), (II) and (III) are TRUE. B) Only (I) is TRUE.

C) Only (II) and (III) are TRUE. D) Only (I) and (III) are TRUE.

10. Let $f(x)=x^3+x^2f'(1)+2x\ f''(2)+f'''(3), x\in R$. Then the value of $f'(5)$ is :

A) $\frac{62}{5}$ B) $\frac{657}{5}$ C) $\frac{2}{5}$ D) $\frac{117}{5}$

11. Let $[\bullet]$ denote the greatest integer function, and let $f(x)=\min\left\{\sqrt{2}x,x^2\right\}$. Let $S=\{x\in(-2,2)$: the function $g(x)=|x|\left[x^2\right]$ is discontinuous at x }. Then $\sum_{x\in S}f(x)$ equals :

A) $2-\sqrt{2}$ B) $2\sqrt{6}-3\sqrt{2}$ C) $1-\sqrt{2}$ D) $\sqrt{6}-2\sqrt{2}$

12. If $f(x)=\begin{cases}\frac{a|x|+x^2-2(\sin|x|)(\cos|x|)}{x} & ,x\neq 0\\ b & ,x=0\end{cases}$ is continuous at $x=0$ then $a+b$ is equal to:

A) 1 B) 2 C) 0 D) 4

13. If $y(x)=\begin{vmatrix}\sin x & \cos x & \sin x+\cos x+1\\ 27 & 28 & 27\\ 1 & 1 & 1\end{vmatrix}, x\in\mathbb{R}$, then $\frac{d^2y}{dx^2}+y$ is equal to

A) -1 B) 28 C) 27 D) 1

14. Let $f:\mathbf{R}\rightarrow\mathbf{R}$ be a twice differentiable function such that $(\sin x\cos y)(f(2x+2y)-f(2x-2y))=(\cos x\sin y)(f(2x+2y)+f(2x-2y))$, for all $x, y\in\mathbf{R}$.
If $f'(0)=\frac{1}{2}$, then the value of $24f''\left(\frac{5\pi}{3}\right)$ is:

A) 2 B) -3 C) 3 D) -2

15. Let f be a real valued continuous function defined on the positive real axis such that $g(x)=\int_0^x tf(t)dt$. If $g\left(x^3\right)=x^6+x^7$, then value of $\sum_{r=1}^{15}f\left(r^3\right)$ is :

A) 270 B) 340 C) 320 D) 310

16. Let $f:\mathbf{R}\rightarrow\mathbf{R}$ be a polynomial function of degree four having extreme values at $x=4$ and $x=5$. If $\lim_{x\rightarrow 0}\frac{f(x)}{x^2}=5$, then $f(2)$ is equal to :

A) 12 B) 10 C) 8 D) 14

17. Let the function $f(x)=\left(x^2+1\right)\left|x^2-ax+2\right|+\cos|x|$ be not differentiable at the two points $x=\alpha=2$ and $x=\beta$. Then the distance of the point (α,β) from the line $12x+5y+10=0$ is equal to :

A) 5 B) 4 C) 3 D) 2

18. Let $[x]$ denote the greatest integer function, and let m and n respectively be the numbers of the points, where the function $f(x)=[x]+|x-2|,-2<x<3$, is not continuous and not differentiable. Then $\mathrm{m}+\mathrm{n}$ is equal to :

A) 6 B) 8 C) 9 D) 7

19. Let $f(\mathrm{x})=\begin{cases}(1+\mathrm{ax})^{1/\mathrm{x}} & , \quad \mathrm{x}<0 \\ 1+\mathrm{b} & , \quad \mathrm{x}=0 \\ \frac{(\mathrm{x}+4)^{1/2}-2}{(\mathrm{x}+\mathrm{c})^{1/3}-2} & ,\end{cases}$

be continuous at $x=0$. Then $e^{a}bc$ is equal to

A) 64 B) 72 C) 48 D) 36

20. If the function

$$f(x)=\begin{cases}\frac{2}{x}\left\{\sin\left(k_1+1\right)x+\sin\left(k_2-1\right)x\right\}, & x<0 \\ 4, \quad x=0 \\ \frac{2}{x}\log_e\left(\frac{2+k_1x}{2+k_2x}\right), & x>0\end{cases}$$

is continuous at $\mathrm{x}=0$, then $\mathrm{k}_1^2+\mathrm{k}_2^2$ is equal to

A) 20 B) 5 C) 8 D) 10

21. If $\log_e y=3\sin^{-1}x$, then $(1-x)^2y''-xy'$ at $x=\frac{1}{2}$ is equal to :

A) $9e^{\pi/6}$ B) $3e^{\pi/6}$ C) $3e^{\pi/2}$ D) $9e^{\pi/2}$

22. Let $\int_0^x\sqrt{1-(y'(t))^2}dt=\int_0^x y(t)dt, 0\leq x\leq 3, y\geq 0$, $\mathrm{y}(0)=0$. Then at $\mathrm{x}=2, \mathrm{y}''+\mathrm{y}+1$ is equal to :

A) 1 B) 2 C) $\sqrt{2}$ D) $1/2$

23. Let $f(x)=ax^3+bx^2+cx+41$ be such that $f(1)=40, f'(1)=2$ and $f''(1)=4$. Then $a^2+b^2+c^2$ is equal to :

A) 62 B) 73 C) 54 D) 51

24. For $a,b>0,$ let $f(x)=\begin{cases}\frac{\tan((a+1)x)+b\tan x}{x}, & x<0 \\ 3 & x=0 \\ \frac{\sqrt{ax+b^2x^2}-\sqrt{ax}}{b\sqrt{ax}\sqrt{x}}, & x>0\end{cases}$ be a continous function at x = 0. Then $\frac{b}{a}$ is equal to

A) 5 B) 4 C) 8 D) 6

25. Suppose for a differentiable function $h, h(0)=0$, $\mathrm{h}(1)=1$ and $\mathrm{h}'(0)=\mathrm{h}'(1)=2$. If $\mathrm{g}(\mathrm{x})=\mathrm{h}\left(\mathrm{e}^{\mathrm{x}}\right)\mathrm{e}^{\mathrm{h}(\mathrm{x})}$, then $g'(0)$ is equal to :

A) 5 B) 3 C) 8 D) 4

26. Let $f:(-\infty,\infty)-\{0\}\rightarrow R$ be a differentiable function such that $f'(1)=\lim_{a\rightarrow\infty}a^2f\left(\frac{1}{a}\right)$.
Then $\lim_{a\rightarrow\infty}\frac{a(a+1)}{2}\tan^{-1}\left(\frac{1}{a}\right)+a^2-2\log_e a$ is equal to

A) $\frac{3}{2}+\frac{\pi}{4}$ B) $\frac{3}{8}+\frac{\pi}{4}$ C) $\frac{5}{2}+\frac{\pi}{8}$ D) $\frac{3}{4}+\frac{\pi}{8}$

27. If $f(x)=\begin{cases}x^3\sin\left(\frac{1}{x}\right), & x\neq 0 \\ 0, & x=0\end{cases}$, then

A) $f''(0)=1$ B) $\mathrm{f}''\left(\frac{2}{\pi}\right)=\frac{24-\pi^2}{2\pi}$ C) $f''\left(\frac{2}{\pi}\right)=\frac{12-\pi^2}{2\pi}$ D) $f''(0)=0$

28. If $y(\theta)=\frac{2\cos\theta+\cos 2\theta}{\cos 3\theta+4\cos 2\theta+5\cos\theta+2}$ then at $\theta=\frac{\pi}{2}, y''+y'+y$ is equal to :

A) $\frac{3}{2}$ B) 1 C) $\frac{1}{2}$ D) 2

29. Let $f:[-1,2]\rightarrow\mathrm{R}$ be given by $f(x)=2x^2+x+\left[x^2\right]-[x]$, where $[t]$ denotes the greatest integer less than or equal to t. The number of points, where f is not continuous, is :

A) 6 B) 3 C) 4 D) 5

30. Let $f(x)=x^5+2x^3+3x+1, x\in R$, and $g(x)$ be a function such that $g(f(x))=x$ for all $x\in R$. Then $\frac{g(7)}{g'(7)}$ is equal to :

A) 7 B) 42 C) 1 D) 14

31. If the function $f(x)=\frac{\sin 3x+\alpha\sin x-\beta\cos 3x}{x^3}, x\in R$, is continuous at $x=0$, then $f(0)$ is equal to :

A) 2 B) -2 C) 4 D) -4

32. If the function $f(x)=\begin{cases}\frac{72^x-9^x-8^x+1}{\sqrt{2}-\sqrt{1+\cos x}} & ,x\neq 0\\ a\log_e 2\log_e 3 & ,x=0\end{cases}$ is continuous at $x=0$, then the value of a^2 is equal to

A) 968 B) 1152 C) 746 D) 1250

33. Let $f(\mathrm{x})=\mathrm{x}^5+2\mathrm{e}^{\mathrm{x}/4}$ for all $\mathrm{x}\in\mathrm{R}$. Consider a function $g(x)$ such that $(gof)(x)=x$ for all $x\in R$. Then the value of $8g'(2)$ is :

A) 16 B) 4 C) 8 D) 2

34. Let $f:\mathrm{R}\to\mathrm{R}$ be a function given by

$$f(x)=\begin{cases}\frac{1-\cos 2x}{x^2} & ,x<0\\ \alpha & ,x=0,\ \text{where } \alpha,\beta\in R. \text{ If}\\ \frac{\beta\sqrt{1-\cos x}}{x} & ,x>0\end{cases}$$

f is continuous at $\mathrm{x}=0$, then $\alpha^2+\beta^2$ is equal to :

A) 48 B) 12 C) 3 D) 6

35. Let $f(\mathrm{x})=\left|2\mathrm{x}^2+5\right|\mathrm{x}|-3|, \mathrm{x}\in\mathrm{R}$. If m and n denote the number of points where f is not continuous and not differentiable respectively, then $m+n$ is equal to :

A) 5 B) 2 C) 0 D) 3

36. Let $f:R\to R$ be defined as $f(x)=\begin{cases}\frac{a-b\cos 2x}{x^2} & ; & x<0\\ x^2+cx+2 & ; & 0\leq x\leq 1\\ 2x+1 & ; & x>1\end{cases}$ If f is continuous everywhere in R and m is the number of points where f is NOT differential then $\mathrm{m}+\mathrm{a}+\mathrm{b}+\mathrm{c}$ equals :

A) 1 B) 4 C) 3 D) 2

37. Consider the function $f:(0,\infty)\to R$ defined by $f(x)=e^{-|\log_e x|}$. If m and n be respectively the number of points at which f is not continuous and f is not differentiable, then $\mathrm{m}+\mathrm{n}$ is

A) 0 B) 3 C) 1 D) 2

38. Let $\mathrm{g(x)}$ be a linear function and $f(x)=\begin{cases}g(x) & ,x\leq 0\\ \left(\frac{1+x}{2+x}\right)^{\frac{1}{x}} & ,x>0\end{cases}$, is continuous at $x=0$. If $f'(1)=f(-1)$, then the value of $g(3)$ is

A) $\frac{1}{3}\log_e\left(\frac{4}{9e^{1/3}}\right)$ B) $\frac{1}{3}\log_e\left(\frac{4}{9}\right)+1$ C) $\log_e\left(\frac{4}{9}\right)-1$ D) $\log_e\left(\frac{4}{9e^{1/3}}\right)$

39. Let $\mathrm{f}:\mathbb{R}-\{0\}\to\mathbb{R}$ be a function satisfying $f\left(\frac{x}{y}\right)=\frac{f(x)}{f(y)}$ for all $x,y,f(y)\neq 0$. If $f'(1)=2024$ then

A) $\mathrm{xf'(x)-2024f(x)=0}$ B) $xf'(x)-2024f(x)=0$

C) $\mathrm{xf'(x)+f(x)=2024}$ D) $xf'(x)-2023f(x)=0$

Maths - Section B (Numeric)

40. If $y=\frac{(\sqrt{x}+1)(x^2-\sqrt{x})}{x\sqrt{x}+x+\sqrt{x}}+\frac{1}{15}\left(3\cos^2 x-5\right)\cos^3 x$, then $96y'\left(\frac{\pi}{6}\right)$ is equal to :

41. Let $\mathrm{f}:\mathbb{R}\to\mathbb{R}$ be a thrice differentiable function such that $f(0)=0, f(1)=1, f(2)=-1, f(3)=2$ and $f(4)=-2$. Then, the minimum number of zeros of $(3f'f''+ff''')(x)$ is ....

42. Let $[t]$ denote the greatest integer less than or equal to t. Let $\mathrm{f}:[0, \infty) \rightarrow \mathrm{R}$ be a function defined by $f(x) = \left[\frac{x}{2}+3\right]-[\sqrt{x}]$. Let S be the set of all points in the interval $[0,8]$ at which f is not continuous. Then $\sum_{\mathrm{a} \in \mathrm{S}} \mathrm{a}$ is equal to ....

43. Lef $f:(0, \pi) \rightarrow R$ be a function given by

$$f(x)=\begin{cases}\left(\frac{8}{7}\right)^{\frac{\tan 8x}{\tan 7x}}, & 0<x<\frac{\pi}{2} \\ a-8, & x=\frac{\pi}{2} \\ (1+|\cot x)^{\frac{b}{a}|\tan x|}, & \frac{\pi}{2}<x<\pi\end{cases}$$

Where $a, b \in Z$. If f is continuous at $x=\frac{\pi}{2}$, then $\mathrm{a}^2+\mathrm{b}^2$ is equal to ....

44. Let

$$\mathrm{f}(x)=\begin{cases}3x, & x<0 \\ \min \{1+x+[x], x+2[x]\}, & 0 \leq x \leq 2 \\ 5, & x>2,\end{cases}$$

where [.] denotes greatest integer function. If α and β are the number of points, where f is not continuous and is not differentiable, respectively, then $\alpha+\beta$ equals _________

45. Let m and n be the number of points at which the function $f(\mathrm{x})=\max \left\{\mathrm{x}, \mathrm{x}^3, \mathrm{x}^5, \ldots ., \mathrm{x}^{21}\right\}, \mathrm{x} \in \mathbb{R}$, is not differentiable and not continuous, respectively. Then $\mathrm{m}+\mathrm{n}$ is equal to _______ .

46. The number of points, at which the function $f(x)=\max \left\{6x, 2+3x^2\right\}+|x-1|\left|\cos \left|x^2-\frac{1}{4}\right|\right|$, $x \in(-\pi, \pi)$, is not differentiable, is _____.

47. Let $f(x)=\begin{cases}e^{x-1}, & x<0 \\ x^2-5x+6, & x \geq 0\end{cases}$ and $g(x)=f(|x|)+|f(x)|$. If the number of points where g is not continuous and is not differentiable are α and β respectively, then $\alpha+\beta$ is equal to ______

48. The number of points in the interval $[2,4]$, at which the function $f(x)=\left[x^2-x-\frac{1}{2}\right]$, where $[\cdot]$ denotes the greatest integer function, is discontinuous, is _______.

49. Let $f(x)=\begin{cases}x^3+8; & x<0 \\ x^2-4; & x \geq 0\end{cases}$ and $g(x)=\begin{cases}(x-8)^{1/3}; & x<0 \\ (x+4)^{1/2}; & x \geq 0\end{cases}$.
Then the number of points, where the function $g \circ f$ is discontinuous, is __________.

50. The number of points of discontinuity of the function $f(\mathrm{x})=\left[\frac{\mathrm{x}^2}{2}\right]-[\sqrt{\mathrm{x}}], \mathrm{x} \in[0,4]$, where $[\cdot]$ denotes the greatest integer function is ________

Continuity and Differentiability

Subjects : Maths Previous Year Questions

Maths - Section A (MCQ)

1 - C	2 - A	3 - D	4 - C	5 - A	6 - B	7 - D	8 - C	9 - D	10 - D
11 - C	12 - B	13 - A	14 - B	15 - D	16 - B	17 - C	18 - B	19 - C	20 - D
21 - D	22 - A	23 - D	24 - D	25 - D	26 - C	27 - B	28 - D	29 - C	30 - D
31 - D	32 - B	33 - A	34 - B	35 - D	36 - D	37 - C	38 - D	39 - A	

Maths - Section B (Numeric)

40 - 105	41 - 5	42 - 17	43 - 81	44 - 5	45 - 3	46 - 9	47 - 4	48 - 10	49 - 3
50 - 8									