

SECTION -A (Multiple choice questions)

- Which of the following functions from $\mathbb{Z}$ into $\mathbb{Z}$ are bijections?
(a) $f(x) = x^3$ (b) $f(x) = x + 2$ (c) $f(x) = 2x + 1$ (d) $f(x) = x^2 + 1$
- The function $f: \mathbb{R} \rightarrow \mathbb{R}$ defined as $f(x) = x^3$ is:
a) One-on but not onto b) Not one-one but onto
c) Neither one-one nor onto d) One-one and onto
- Let $A = \{1, 2, 3, \dots, n\}$ and $B = \{a, b\}$. Then the number of surjections from A into B is
(a) ${}^n P_2$ (b) $2^n - 2$ (c) $2^n - 1$ (d) none of these
- For real numbers x and y , define xRy if and only if $x - y + \sqrt{2}$ is an irrational number. Then the relation R is
(a) reflexive (b) symmetric (c) transitive (d) none of these
- Set A has 3 elements and the set B has 4 elements. Then the number of injective mappings that can be defined from A to B is
(a) 144 (b) 12 (c) 24 (d) 64
- If a relation R on the set $\{1, 2, 3\}$ be defined by $R = \{(1, 2)\}$, then R is
(a) reflexive (b) transitive (c) symmetric (d) none of these
- Number of reflexive relations are possible in a set A whose $n(A) = 4$
(a) 16 (b) 1024 (c) 2048 (d) 4096
- Let R be a relation defined on $\mathbb{Z}$ as $R = \{(x, y) : |x - y| \leq 1\}$ Then R is:
(a) Reflexive and transitive (b) Reflexive and symmetric
(c) Symmetric and transitive (d) an equivalence relation
- Let S be the set of all real numbers. Then, the relation $R = \{(a, b) : 1 + ab > 0\}$ on S is
(a) Reflexive, symmetric but not transitive (b) Reflexive, transitive but not symmetric
(c) Symmetric, transitive but not reflexive (d) reflexive, transitive and symmetric

10. Let $A = \{x : -1 \leq x \leq 1\}$ and $f : A \rightarrow A$ is a function defined by $f(x) = x |x|$ then f is
- (a) a bijection (b) injection but not surjection
(c) surjection but not injection (d) neither injection nor surjection
11. The function $F: \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = (x-1)(x-2)(x-3)$ is
- (a) one-one but not onto (b) onto but not one-one
(c) both one-one and onto (d) neither one-one and onto

SECTION -B

(Short Answer Type Questions)

12. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined as $f(x) = [x]$. Check whether f is one-one and onto?
13. Let $R = \{(1,3), (3,1), (1,1)\}$ be Relations, Check whether Relation is reflexive, Symmetric or Transitive.
14. A relation R on set $A = \{1, 2, 3, 4, 5\}$ is defined as $R = \{(x, y) : |x^2 - y^2| < 8\}$. Check whether the relation R is reflexive, symmetric, and transitive.
15. Let $A = \{1, 2, 3\}$. Then find the number of relations containing $(1, 2)$ and $(2, 3)$ which are reflexive and transitive but not symmetric.
16. A relation R on $A = \{1, 2, 3\}$ is defined as $R = \{(1, 1), (3, 3), (1, 2)\}$. Is R a symmetric relation? Justify. Write the smallest relation set R_1 such that $R \cup R_1$ becomes an equivalence relation on the set $\{1, 2, 3\}$.
17. Let R be the relation defined in the set $A = \{1, 2, 3, 4, 5, 6, 7\}$ by $R = \{(a, b) : \text{both } a \text{ and } b \text{ are either odd or even}\}$. Show that R is an equivalence relation. Hence, find the elements of equivalence class $[1]$.
18. A function $f : A \rightarrow B$ defined as $f(x) = 2x$ is both one-one and onto. If $A = \{1, 2, 3, 4\}$, then find the set B .
19. A relation R is defined on a set of real numbers as $R = \{(x, y) : x \cdot y \text{ is an irrational number}\}$. Check whether R is reflexive, symmetric and transitive or not.

SECTION -C

(Long Answer Type Questions)

20. Show that the relation S in the set $A = \{x \in \mathbb{Z} : 0 \leq x \leq 12\}$ given by ; $S = \{(a, b) : a, b \in \mathbb{Z}, |a-b| \text{ is divisible by } 4\}$ is an equivalence relation, Find the set of elements related to 1.

21. Let S is the set of all real numbers & R is the relation in S , defined by $R = \{(a, b) : a \leq b^3\}$.
Show that R satisfies none of reflexivity, symmetry & transitivity.
22. let $A = \{1, 2, 3, \dots, 10\}$ and R be the relation in A defined by $(a, b) R (c, d)$ iff $a + d = b + c$ for all (a, b) and $(c, d) \in A \times A$. Prove that R is an equivalence relation. Also obtain the equivalence class $[(2, 5)]$.
23. Let N denote the set of all natural numbers and R be the relation on $N \times N$ defined by $(a, b) R (c, d) \Leftrightarrow a d (b + c) = b c (a + d)$. Check whether R is an equivalence relation. Also, find the equivalence class of $[(2, 6)]$.
24. If $A = \mathbb{R} - \{3\}$ and $B = \mathbb{R} - \{1\}$. Consider the function $f : A \rightarrow B$ defined by $f(x) = \frac{x-2}{x-3}$ for all $x \in A$.
Then, show that f is bijective.
25. Show that a function $f : \mathbb{R}_+ \rightarrow A \subset \mathbb{N}$, defined as $f(x) = 4x^2 + 12x + 15$ is one-one. Find set A so that f is onto where $\mathbb{R}_+ = [0, \infty)$. Also, find if there exists $a \in \mathbb{R}_+$ such that $f(a) = 7$. Justify.
26. Show that function $f : \mathbb{R} \rightarrow \{x \in \mathbb{R} : -1 < x < 1\}$ defined by $f(x) = \frac{x}{1+|x|}$, $x \in \mathbb{R}$ is one one & onto functions.
27. Show that the function that $f : \mathbb{N} \rightarrow \mathbb{N}$ given by : $f(n) = n - (-1)^n$ for all $n \in \mathbb{N}$ is a bijection.
28. A school wants the students of class XII to do a project on 'Sustainability' keeping the world environment in mind. They select the student participants on the basis of an essay writing competition. 7 students out of 80 are selected for the project and are categorized into two sets such that : Girl students belong to Set $A = \{G_1, G_2, G_3, G_4\}$, Boy students belong to Set $B = \{B_1, B_2, B_3\}$. Based on the above information, answer the following questions:
- How many relations are possible from Set $A \rightarrow$ Set B ?
 - Let R be a relation from $A \rightarrow B$ such that $R = \{(G_1, B_1), (G_2, B_2), (G_3, B_2), (G_4, B_3), (G_1, B_2)\}$. Is R an injective function ? Justify your answer.
 - Let the relation R from $A \rightarrow A$ be such that $R = \{(x, y), x, y \in A, x \text{ and } y \text{ are students from the same colony in the city}\}$ Verify if R is an equivalence relation.
 - Verify if any function $f : B \rightarrow A$ is bijective. Give reason to support your answer.

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Relations and Functions- Answer Key

SECTION A

1. (b) $f(x) = x + 2$
2. (d) One-one and onto
3. (b) $2^n - 2$
4. (a) reflexive
5. (c) 24
6. (b) transitive
7. (d) 4096
8. (b) Reflexive and symmetric
9. (a) Reflexive, symmetric but not transitive
10. (a) a bijection
11. (b) onto but not one-one

SECTION B

12. Neither one-one nor onto
13. Symmetric only (not reflexive, not transitive)
14. Reflexive and symmetric, not transitive
15. 3
16. Not symmetric. $R_1 = \{(2,2), (2,1)\}$
17. Equivalence relation. Equivalence class $[1] = \{1, 3, 5, 7\}$
18. $B = \{2, 4, 6, 8\}$
19. Not reflexive, symmetric, not transitive

SECTION C

20. Equivalence relation. Set related to 1 = $\{1, 5, 9\}$
21. Neither reflexive, nor symmetric, nor transitive
22. Equivalence relation. $[(2,5)] = \{(1,4), (2,5), (3,6), (4,7), (5,8), (6,9), (7,10)\}$

23. Equivalence relation. $[(2,6)] = \{(2,6)\}$

24. Bijective

25. One-one. $A = [15, \infty)$ No such a exists

26. One-one and onto (bijective)

27. Bijection

28. (i) 4096

(ii) Not injective (not even a function)

(iii)(a) Equivalence relation (b) No function $f: B \rightarrow A$ is bijective

