Tarun Classes of Mathematics

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Applications of Derivatives

Subjects : Maths **Previous Year Questions**

Maths - Section A (MCQ)

1. The number of critical points of the function $f(x) = \begin{cases} \left|\dfrac{\sin x}{x}\right|, & x \neq 0 \\ 1, & x = 0 \end{cases}$ in the interval $(-2\pi, 2\pi)$ is equal to :

A) 1 **B)** 3 **C)** 5 **D)** 7

2. Let $f(x)$ be a polynomial of degree 5, and have extrema at $x = 1$ and $x = -1$. If $\lim\limits_{x \to 0}\left(\dfrac{f(x)}{x^3}\right) = -5$, then $f(2)$ $- f(-2)$ is equal to:

A) 0 **B)** 50 **C)** 92 **D)** 112

3. $\max_{0 \leq x \leq \pi}\left(16 \sin\left(\dfrac{x}{2}\right)\cos^3\left(\dfrac{x}{2}\right)\right)$ is equal to:

A) $\dfrac{3\sqrt{3}}{2}$ **B)** $3\sqrt{3}$ **C)** $4\sqrt{3}$ **D)** $6\sqrt{3}$

4. Let $f : \mathbb{R} \to \mathbb{R}$ be a differentiable function such that $f\left(\dfrac{x+y}{3}\right) = \dfrac{f(x) + f(y)}{3}$ for all $x, y \in \mathbb{R}$, and $f'(0) = 3$. Then the minimum value of the function $g(x) = 3 + e^x f(x)$, is:

A) $3\left(\dfrac{e+1}{e}\right)$ **B)** $3\left(\dfrac{e-1}{e}\right)$ **C)** $\dfrac{3-e}{e}$ **D)** $3e$

5. Let $f(x)$ and $g(x)$ be twice differentiable functions satisfying $f''(x) = g''(x)$ for all $x \in \mathbf{R}$, $f'(1) = 2g'(1) = 4$ and $g(2) = 3f(2) = 9$. Then $f(25) - g(25)$ is equal to :

A) 20 **B)** 40 **C)** -20 **D)** -40

6. Let $f : R \to R$ be a twice differentiable function such that $f''(x) > 0$ for all $x \in R$ and $f'(a-1) = 0$, where a is real number. Let $g(x) = f(tan^2 x - 2\ tan\ x + a), 0 < x < \frac{\pi}{2}$.
Consider the following two statements :
(I) g is increasing in $(0, \frac{\pi}{4})$
(II) g is decreasing in $(\frac{\pi}{4}, \frac{\pi}{2})$
Then,

A) Neither (I) nor (II) is True **B)** Only (II) is True

C) Only (I) is True **D)** Both (I) and (II) are True

7. Let $f(x) = x^{2025} - x^{2000}, x \in [0, 1]$and the minimum value of the function $f(x)$ in the interval [0, 1] be $(80)^{80}(n)^{-81}$. Then n is equal to

A) -81 **B)** -40 **C)** -41 **D)** -80

8. Let the length of a latus rectum of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ be 10 . If its eccentricity is the minimum value of the function $f(\mathrm{t}) = \mathrm{t}^2 + \mathrm{t} + \frac{11}{12}$, $\mathrm{t} \in \mathbf{R}$, then $\mathrm{a}^2 + \mathrm{b}^2$ is equal to :

A) 125 **B)** 126 **C)** 120 **D)** 115

9. Let $x=-1$ and $x=2$ be the critical points of the function $\mathrm{f}(\mathrm{x})=\mathrm{x}^3+\mathrm{ax}^2+\mathrm{b}\log_{\mathrm{c}}|\mathrm{x}|+1, \mathrm{x}\neq 0$. Let m and M respectively be the absolute minimum and the absolute maximum values of f in the interval $\left[-2,-\frac{1}{2}\right]$. Then $|\mathrm{M}+m|$ is equal to (Take $\log_{\mathrm{c}} 2=0.7$):

A) 21.1 B) 19.8 C) 22.1 D) 20.9

10. Let $\mathrm{a}>0$. If the function $\mathrm{f}(\mathrm{x})=6\mathrm{x}^3-45\mathrm{ax}^2+108\mathrm{a}^2\mathrm{x}+1$ attains its local maximum and minimum values at the points x_1 and x_2 respectively such that $x_1 x_2=54$, then $\mathrm{a}+\mathrm{x}_1+\mathrm{x}_2$ is equal to :-

A) 15 B) 18 C) 24 D) 13

11. Let $\mathrm{f}:\mathrm{R}\rightarrow\mathrm{R}$ be a function defined by $f(x)=\| x+2|-2|x\|$. If m is the number of points of local minima and n is the number of points of local maxima of f, then $m+n$ is

A) 5 B) 3 C) 2 D) 4

12. If the function $f(x)=2x^3-9\mathrm{ax}^2+12\mathrm{a}^2\mathrm{x}+1$, where $\mathrm{a}>0$, attains its local maximum and local minimum values at p and q, respectively, such that $\mathrm{p}^2=\mathrm{q}$, then $f(3)$ is equal to:

A) 55 B) 10 C) 23 D) 37

13. The sum of all local minimum values of the function is

$$f(x)=\begin{cases} 1-2x, & x<-1 \\ \frac{1}{3}(7+2|x|), & -1\leq x\leq 2 \\ \frac{11}{18}(x-4)(x-5), & x>2 \end{cases}$$

A) $\frac{157}{72}$ B) $\frac{131}{72}$ C) $\frac{171}{72}$ D) $\frac{167}{72}$

14. Let $f(x)=\int_0^{x^2}\frac{\mathrm{t}^2-8\mathrm{t}+15}{\mathrm{e}^{\mathrm{t}}}\mathrm{dt}, x\in\mathbf{R}$. Then the numbers of local maximum and local minimum points of f, respectively, are :

A) 2 and 3 B) 2 and 2 C) 3 and 2 D) 1 and 3

15. Let the function $f(x)=\frac{x}{3}+\frac{3}{x}+3, x\neq 0$ be strictly increasing in $(-\infty,\alpha_1)\,\mathrm{U}\,(\alpha_2,\infty)$ and strictly decreasing in $(\alpha_3,\alpha_4)\,\mathrm{U}\,(\alpha_4,\alpha_5)$. Then $\sum_{\mathrm{i}=1}^{5}\alpha_{\mathrm{i}}^2$ is equal to :-

A) 48 B) 28 C) 40 D) 36

16. Let $(2,3)$ be the largest open interval in which the function $f(x)=2\log_{\mathrm{e}}(x-2)-x^2+ax+1$ is strictly increasing and (b, c) be the largest open interval, in which the function $\mathrm{g}(x)=(x-1)^3(x+2-\mathrm{a})^2$ is strictly decreasing. Then $100(a+b-c)$ is equal to :

A) 420 B) 360 C) 160 D) 280

17. A spherical chocolate ball has a layer of ice-cream of uniform thickness around it. When the thickness of the ice-cream layer is 1 cm , the ice-cream melts at the rate of $81\ \mathrm{cm}^3/\mathrm{min}$ and the thickness of the ice-cream layer decreases at the rate of $\frac{1}{4\pi}\ \mathrm{cm}/\mathrm{min}$. The surface area (in cm^2) of the chocolate ball (without the ice-cream layer) is :

A) 196π B) 256π C) 225π D) 128π

18. For the function $\mathrm{f}(\mathrm{x})=(\cos\mathrm{x})-\mathrm{x}+1, \mathrm{x}\in\mathbb{R}$, between the following two statements
($S1$) $f(x)=0$ for only one value of x is $[0,\pi]$.
($S2$) $\mathrm{f}(\mathrm{x})$ is decreasing in $\left[0,\frac{\pi}{2}\right]$ and increasing in $\left[\frac{\pi}{2},\pi\right]$.

A) Both ($S1$) and ($S2$) are correct B) Only ($S1$) is correct

C) Both ($S1$) and ($S2$) are incorrect D) Only ($S2$) is correct

19. If the function $f(x)=2x^3-9ax^2+12a^2x+1, a>0$ has a local maximum at $\mathrm{x}=\alpha$ and a local minimum $x=\alpha^2$, then α and α^2 are the roots of the equation :

A) $\mathrm{x}^2-6\mathrm{x}+8=0$ B) $8x^2+6x-8=0$ C) $8x^2-6x+1=0$ D) $x^2+6x+8=0$

20. Let $f(x)=4\cos^3 x+3\sqrt{3}\cos^2 x-10$. The number of points of local maxima of f in interval $(0,2\pi)$ is:

A) 1 B) 2 C) 3 D) 4

21. The number of critical points of the function $f(x)=(x-2)^{2/3}(2x+1)$ is :

A) 2 B) 0 C) 1 D) 3

22. If the function $f(x)=\left(\frac{1}{x}\right)^{2x}; x>0$ attains the maximum value at $\mathrm{x}=\frac{1}{\mathrm{e}}$ then :

A) $\mathrm{e}^{\pi}<\pi^{\mathrm{e}}$ B) $\mathrm{e}^{2\pi}<(2\pi)^{\mathrm{e}}$ C) $\mathrm{e}^{\pi}>\pi^{\mathrm{e}}$ D) $(2\mathrm{e})^{\pi}>\pi^{(2\mathrm{e})}$

23. The interval in which the function $\mathrm{f}(\mathrm{x})=\mathrm{x}^{\mathrm{x}}, \mathrm{x}>0$, is strictly increasing is

A) $\left(0,\frac{1}{\mathrm{e}}\right]$ B) $\left[\frac{1}{\mathrm{e}^2},1\right)$ C) $(0,\infty)$ D) $\left[\frac{1}{\mathrm{e}},\infty\right)$

24. For the function
$f(x)=\sin x+3x-\frac{2}{\pi}\left(x^2+x\right)$, where $x\in\left[0,\frac{\pi}{2}\right]$,
consider the following two statements :
(I) f is increasing in $\left(0,\frac{\pi}{2}\right)$.
(II) f' is decreasing in $\left(0,\frac{\pi}{2}\right)$.
Between the above two statements,

A) only (I) is true. B) only (II) is true.

C) neither (I) nor (II) is true . D) both (I) and (II) are true.

25. Let $f(x)=3\sqrt{x-2}+\sqrt{4-x}$ be a real valued function. If α and β are respectively the minimum and the maximum values of f, then $\alpha^2+2\beta^2$ is equal to

A) 44 B) 42 C) 24 D) 38

26. If $5f(x)+4f\left(\frac{1}{x}\right)=x^2-2, \forall x\neq 0$ and $y=9x^2 f(x)$, then y is strictly increasing in :

A) $\left(0,\frac{1}{\sqrt{5}}\right)\cup\left(\frac{1}{\sqrt{5}},\infty\right)$ B) $\left(-\frac{1}{\sqrt{5}},0\right)\cup\left(\frac{1}{\sqrt{5}},\infty\right)$

C) $\left(-\frac{1}{\sqrt{5}},0\right)\cup\left(0,\frac{1}{\sqrt{5}}\right)$ D) $\left(-\infty,\frac{1}{\sqrt{5}}\right)\cup\left(0,\frac{1}{\sqrt{5}}\right)$

27. If the function $f:(-\infty,-1]\rightarrow(a,b]$ defined by $f(x)=e^{x^3-3x+1}$ is one-one and onto, then the distance of the point $\mathrm{P}(2\mathrm{~b}+4,\mathrm{a}+2)$ from the line $x+e^{-3}y=4$ is :

A) $2\sqrt{1+\mathrm{e}^6}$ B) $4\sqrt{1+\mathrm{e}^6}$ C) $3\sqrt{1+\mathrm{e}^6}$ D) $\sqrt{1+\mathrm{e}^6}$

28. Let $f:\rightarrow R\rightarrow(0,\infty)$ be strictly increasing function such that $\lim_{x\rightarrow\infty}\frac{f(7x)}{f(x)}=1$. Then, the value of $\lim_{x\rightarrow\infty}\left[\frac{f(5x)}{f(x)}-1\right]$ is equal to

A) 4 B) 0 C) $7/5$ D) 1

29. Let a variable line passing through the centre of the circle $x^2+y^2-16x-4y=0$, meet the positive co-ordinate axes at the point A and B. Then the minimum value of $\mathrm{OA}+\mathrm{OB}$, where O is the origin, is equal to

A) 12 B) 18 C) 20 D) 24

30. Let $f(x)=(x+3)^2(x-2)^3, x\in[-4,4]$. If M and m are the maximum and minimum values of f, respectively in $[-4,4]$, then the value of $M-m$ is :

A) 600 B) 392 C) 608 D) 108

31. The maximum area of a triangle whose one vertex is at $(0,0)$ and the other two vertices lie on the curve $y=-2x^2+54$ at points (x,y) and $(-x,y)$ where $\mathrm{y}>0$ is :

A) 88 B) 122 C) 92 D) 108

32. The function $f(x)=\frac{x}{x^2-6x-16}, x\in\mathbb{R}-\{-2,8\}$

A) decreases in $(-2,8)$ and increases in $(-\infty,-2)\cup(8,\infty)$

B) decreases in $(-\infty,-2)\cup(-2,8)\cup(8,\infty)$

C) decreases in $(-\infty,-2)$ and increases in $(8,\infty)$

D) increases in $(-\infty,-2)\cup(-2,8)\cup(8,\infty)$

33. The function $f(x)=2x+3(x)^{\frac{2}{3}}, x\in\mathbb{R}$, has

A) exactly one point of local minima and no point of local maxima

B) exactly one point of local maxima and no point of local minima

C) exactly one point of local maxima and exactly one point of local minima

D) exactly two points of local maxima and exactly one point of local minima

34. In an $A.P.$, the sixth terms $a_6=2$. If the $a_1a_4a_5$ is the greatest, then the common difference of the $A.P.$, is equal to

A) $\frac{3}{2}$ B) $\frac{3}{2}$ C) $\frac{2}{3}$ D) $\frac{5}{8}$

35. Let $g(x)=3f\left(\frac{x}{3}\right)+f(3-x)$ and $f''(x)>0$ for all $\mathrm{x}\in(0,3)$. If g is decreasing in $(0,\alpha)$ and increasing in $(\alpha,3)$, then 8α is

A) 24 B) 0 C) 18 D) 20

36. $\max_{0\le x\le\pi}\left\{x-2\sin x\cos x+\frac{1}{3}\sin 3x\right\}=$

A) $\frac{5\pi+2+3\sqrt{3}}{6}$ B) $\frac{\pi+2-3\sqrt{3}}{6}$ C) π D) 0

37. If the local maximum value of the function $f(x)=\left(\frac{\sqrt{3e}}{2\sin x}\right)^{\sin^2 x}$, $x\in\left(0,\frac{\pi}{2}\right)$, is $\frac{k}{e}$, then $\left(\frac{k}{e}\right)^8+\frac{k^8}{e^5}+k^8$ is equal to

A) $e^5+e^6+e^{11}$ B) $e^3+e^5+e^{11}$ C) $e^3+e^6+e^{11}$ D) $e^3+e^6+e^{10}$

38. Let $g(x)=f(x)+f(1-x)$ and $f''(x)>0, x\in(0,1)$. If g is decreasing in the interval $(0,\alpha)$ and increasing in the interval $(\alpha,1)$, then $\tan^1(2\alpha)+\tan^{-1}\left(\frac{1}{\alpha}\right)+\tan^{-1}\left(\frac{\alpha+1}{\alpha}\right)$ is equal to :

A) $\frac{3\pi}{2}$ B) π C) $\frac{5\pi}{4}$ D) $\frac{3\pi}{4}$

Maths - Section B (Numeric)

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39. Let quadratic curve passing through the point $(-1,0)$ and touching the line $y=x$ at $(1,1)$ be $y=f(x)$. Then the x-intercept of the normal to the curve at the point $(\alpha,\alpha+1)$ in the first quadrant is ....

40. A square piece of tin of side $30\ cm$ is to be made into a box without top by cutting a square from each corner and folding up the flaps to form a box. If the volume of the box is maximum, then its surface area (in cm^2) is equal to ....

41. If a_n is the greatest term in the sequence $a_n=\frac{n^3}{n^4+147}, n=1,2,3\ldots\ldots$, then α is equal to ....

42. Let a curve $y=f(x), x\in(0,\infty)$ pass through the points $P\left(1,\frac{3}{2}\right)$ and $Q\left(a,\frac{1}{2}\right)$. If the tangent at any point $R(b,f(b))$ to the given curve cuts the y-axis at the point $S(0,c)$ such that $bc=3$, then $(PQ)^2$ is equal to ....

43. The sum of the abosolute maximum and minimum values of the function $f(x)=\left|x^2-5x+6\right|-3x+2$ in the interval $[-1,3]$ is equal to :

44. Let $(2\alpha,\alpha)$ be the largest interval in which the function $f(t)=\frac{|t+1|}{t^2}, t<0$, is strictly decreasing. Then the local maximum value of the function $g(x)=2\log_e(x-2)+\alpha x^2+4x-\alpha, x>2$, is

45. Let f: R→R be a twice differentiable function such that the quadratic equation $f(x)m^2-2f'(x)m+f''(x)=0$ in m, has two equal roots for every $x \in R$. If $f(0)=1, f'(0)=2$ and (α, β) is the largest interval in which the function $f(\log_e x - x)$ is increasing, then $\alpha+\beta$ is equal to:

46. Let the set of all values of p, for which
$f(x)=(p^2-6p+8)(\sin^2 2x-\cos^2 2x)+2(2-p)x+7$ does not have any critical point, be the interval (a, b). Then $16ab$ is equal to ....

47. Let the set of all positive values of λ, for which the point of local minimum of the function $(1+x(\lambda^2-x^2))$ satisfies $\frac{x^2+x+2}{x^2+5x+6}<0$, be (α, β). Then $\alpha^2+\beta^2$ is equal to ....

48. Let A be the region enclosed by the parabola $y^2=2x$ and the line $x=24$. Then the maximum area of the rectangle inscribed in the region A is ....

49. Let the maximum and minimum values of $\left(\sqrt{8x-x^2-12}-4\right)^2+(x-7)^2, x \in R$ be M and m respectively. Then $\mathrm{M}^2-\mathrm{m}^2$ is equal to ....

50. Let $S=(-1, \infty)$ and $f: S \rightarrow \mathbb{R}$ be defined as $f(x)=\int_{-1}^{x}\left(e^1-1\right)^{11}(2t-1)^5(t-2)^7(t-3)^{12}(2t-10)^{61} dt$ Let $p=$ Sum of square of the values of x, where f(x) attains local maxima on S. and q = Sum of the values of x, where $f(x)$ attains local minima on S. Then, the value of p^2+2q is

Applications of Derivatives

Subjects : Maths Previous Year Questions

Maths - Section A (MCQ)

1 - C	2 - D	3 - B	4 - B	5 - B	6 - A	7 - A	8 - B	9 - A	10 - B
11 - B	12 - D	13 - A	14 - A	15 - D	16 - B	17 - B	18 - B	19 - A	20 - B
21 - A	22 - C	23 - D	24 - D	25 - B	26 - B	27 - A	28 - B	29 - B	30 - C
31 - D	32 - B	33 - C	34 - B	35 - C	36 - A	37 - C	38 - B		

Maths - Section B (Numeric)

39 - 11	40 - 800	41 - 5	42 - 5	43 - 10	44 - 4	45 - 1	46 - 252	47 - 39	48 - 128
49 - 1600	50 - 27								